

GPD and PDF modeling in terms of effective light-cone wave functions

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Abstract. We employ models from effective two-body light-cone wave functions (LCWFs) to provide a link between generalized parton distributions (GPDs) and unintegrated parton distribution functions (uPDFs). Since we utilize the underlying Lorentz symmetry, GPDs can be entirely obtained from the parton number conserved LCWF overlaps. This also allows us to derive model constraints among GPDs. We illustrate that transversity distributions may be rather sizeable.

Keywords: GPDs, unintegrated PDFs, overlap representation of LCWFs, spectator quark model

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INTRODUCTION

In the field theoretical study of the hadron structure the light-cone Hamiltonian approach [1, 2, 3] is rather inspiring, since it describes a hadron in terms of partonic, i.e., quark and gluon, degrees of freedom. Thereby, the LCWFs $\psi_n^S(X_i, \mathbf{k}_{\perp i}, s_i)$ are the probability amplitudes for their corresponding n -parton states $|n, p_i^+, \mathbf{k}_{\perp i}, s_i\rangle$, which build up the proton state with spin projection $S = \{+1/2(\Rightarrow), -1/2(\Leftarrow)\}$ along the z -axis:

$$|P, S\rangle = \sum_n \int [dX d^2\mathbf{k}_{\perp}]_n \prod_{j=1}^n \frac{1}{\sqrt{X_j}} \psi_n^S(X_i, \mathbf{k}_{\perp i}, s_i) |n, X_i P^+, X_i \mathbf{P}_{\perp} + \mathbf{k}_{\perp i}, s_i\rangle. \quad (1)$$

Here, the parton i has longitudinal momentum fraction X_i , transverse momentum $X_i \mathbf{P}_{\perp} + \mathbf{k}_{\perp i}$, spin projection s_i , and $[dX d^2\mathbf{k}_{\perp}]_n$ denotes the n -parton phase space, cf. [4]. The LCWFs have been used for the investigation of the hadron properties [3] and the LCWF overlap representation of GPDs was studied in [5, 6, 7, 4].

When we consider a scalar-diquark spectator model, we have only four LCWFs [4],

$$\Psi_{\rightarrow}^{\Rightarrow} = \frac{m + XM}{M\sqrt{1-X}} \sqrt{\rho(\lambda)} \phi(X, \mathbf{k}_{\perp}|\lambda), \quad \Psi_{\leftarrow}^{\Rightarrow} = \frac{-k^1 - ik^2}{M\sqrt{1-X}} \sqrt{\rho(\lambda)} \phi(X, \mathbf{k}_{\perp}|\lambda), \quad (2)$$

$$\Psi_{\leftarrow}^{\Leftarrow} = \frac{m + XM}{M\sqrt{1-X}} \sqrt{\rho(\lambda)} \phi(X, \mathbf{k}_{\perp}|\lambda), \quad \Psi_{\rightarrow}^{\Leftarrow} = \frac{k^1 - ik^2}{M\sqrt{1-X}} \sqrt{\rho(\lambda)} \phi(X, \mathbf{k}_{\perp}|\lambda), \quad (3)$$

in terms of one *effective* LCWF $\phi(X, \mathbf{k}_{\perp}|\lambda)$, which we weight with the square root of the spectral density $\rho(\lambda)$. Here, λ is the scalar diquark spectator mass, m is the struck

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quark mass, and M is the proton mass. In (2–3) the proton has its spin projection (\Rightarrow) or (\Leftarrow) along the z -axis, and the constituent quark state has the aligned (\rightarrow) or opposite (\leftarrow) spin projection and the orbital angular momentum projection $L^z = 0$ or $L^z = \pm 1$.

LCWF OVERLAP AND GPD CONSTRAINTS

We define in our model the scalar LCWF overlap in the outer region $x \geq \eta$ as

$$\Phi(x \geq \eta, \eta, \Delta_\perp, \mathbf{k}_\perp) = \int d\lambda^2 \frac{\rho(\lambda^2)}{1-x} \phi^*\left(\frac{x-\eta}{1-\eta}, \mathbf{k}_\perp - \frac{1-x}{1-\eta} \Delta_\perp \middle| \lambda\right) \phi\left(\frac{x+\eta}{1+\eta}, \mathbf{k}_\perp \middle| \lambda\right), \quad (4)$$

which represents a unintegrated GPD (uGPD). In the forward limit, i.e., $\Phi(x, \mathbf{k}_\perp) \equiv \Phi(x, \eta = 0, \Delta_\perp = 0, \mathbf{k}_\perp)$, it reduces to a uPDF, and the integration over \mathbf{k}_\perp gives a GPD or a PDF. Since various partonic quantities can be expressed in terms of the LCWF overlap (4), we have their model dependent relations, for example, the relations among the uPDFs which were extensively studied, see, e.g., Ref. [8] and references therein.

To construct the GPD in the central region, i.e., $|x| \leq \eta$, it is useful to employ the double distribution (DD) representation [9, 10], which reads in the unintegrated form as

$$\Phi(x, \eta, \Delta_\perp, \mathbf{k}_\perp) = \int_0^1 dy \int_{-1+y}^{1-y} dz \delta(x-y-z\eta) \int d\lambda^2 \rho(\lambda^2) \hat{\Phi}(y, z, t, \bar{\mathbf{k}}_\perp | \lambda), \quad (5)$$

where $\bar{\mathbf{k}}_\perp = \mathbf{k}_\perp - (1-y+z)\Delta_\perp/2$ and $\hat{\Phi}$ is the Laplace transform of the LCWF overlap,

$$\begin{aligned} \hat{\Phi}(y, z, t, \bar{\mathbf{k}}_\perp | \lambda) &= \frac{1}{2} \int_0^\infty dA A \varphi^*\left(A \frac{1-y+z}{2}\right) \varphi\left(A \frac{1-y-z}{2}\right) \\ &\times \exp\left\{-A \left[(1-y) \frac{m^2}{M^2} + y \frac{\lambda^2}{M^2} - y(1-y) - [(1-y)^2 - z^2] \frac{t}{4M^2} + \frac{\bar{\mathbf{k}}_\perp^2}{M^2}\right]\right\}, \end{aligned} \quad (6)$$

which depends besides the mass parameters also on the functional form of the reduced LCWF $\varphi(\alpha)$. Note that this representation is inspired from a commonly used spectator quark model, which can be expressed in terms of a triangle diagram.

The eight twist-two GPDs $F \in \{H, E, \tilde{H}, \tilde{E}, H_T, E_T, \tilde{H}_T, \tilde{E}_T\}$ can be straightforwardly expressed in terms of the scalar LCWF overlap [4], e.g., in the chiral odd sector:

$$F_T(x, \eta, t) = \int_0^1 dy \int_{-1+y}^{1-y} dz \delta(x-y-\eta z) f_T(x, \eta, t), \quad (7)$$

where the DDs are given in terms of the scalar LCWF overlap (5), integrated over \mathbf{k}_\perp :

$$h_T = \left[\left(\frac{m}{M} + y\right)^2 - ((1-y)^2 - z^2) \frac{t}{4M^2}\right] \hat{\Phi}(y, z, t), \quad \tilde{h}_T = -[(1-y)^2 - z^2] \hat{\Phi}(y, z, t), \quad (8)$$

$$e_T = 2 \left[\left(\frac{m}{M} + y\right) (1-y) + (1-y)^2 - z^2\right] \hat{\Phi}(y, z, t), \quad \tilde{e}_T = 2 \left(\frac{m}{M} + y\right) z \hat{\Phi}(y, z, t). \quad (9)$$

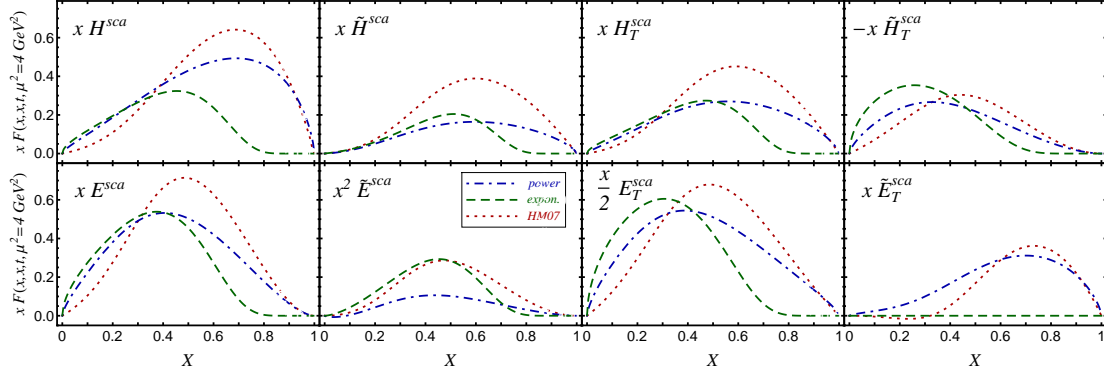


FIGURE 1. Our Regge improved power-like (dash-dotted), exponential (dashed) LCWF, and HM07 (dotted) [4] models are displayed for valence quark GPDs $F^{sca} = 2F^{u_{val}}/3 - F^{d_{val}}/3$ at $\eta = x$ and $t = -0.2 \text{ GeV}^2$ versus $X = 2x/(1+x)$. Upper [lower] panels from left to right: xH [$x\tilde{H}$], \tilde{H} [$x^2\tilde{E}$], H_T [$x\tilde{E}_T/2$], and $-\tilde{H}_T$ [$x\tilde{E}_T$].

The factor $(m/M+y)^2$ in h_T arises from the diagonal $L^z = 0$ LCWF overlap, while the t -dependent part comes from an overlap of $L^z = +1$ and $L^z = -1$ LCWFs ($\Delta L^z = 2$). The \tilde{h}_T DD is entirely induced from a $\Delta L^z = 2$ LCWF overlap and it is related to the so-called "pretzelosity" uPDF. The \tilde{e}_T DD contains a $|L^z| = 1$ and $L^z = 0$ overlap ($\Delta L^z = 1$) and e_T DD contains both $\Delta L^z = 1$ and $\Delta L^z = 2$ overlaps.

Obviously, we can write down various relations among different DDs and express the chiral odd ones by the chiral even ones. Let us consider here only the chiral odd GPDs

$$\overline{H}_T = H_T - \frac{t}{4M^2} \tilde{H}_T, \quad \overline{E}_T = E_T + 2\tilde{H}_T, \quad (10)$$

in which the $\Delta L^z = 2$ overlap is removed. In any scalar diquark model these chiral odd GPDs differ from the chiral even ones by a η proportional term, which is given by \tilde{E}_T :

$$\overline{H}_T(x, \eta, t) \stackrel{\text{mod}}{=} \frac{1}{2} [H + \tilde{H} - \eta \tilde{E}_T](x, \eta, t), \quad \overline{E}_T(x, \eta, t) \stackrel{\text{mod}}{=} E(x, \eta, t) + \eta \tilde{E}_T(x, \eta, t). \quad (11)$$

The chiral odd GPD \tilde{E}_T vanishes by itself in the zero-skewness case, since the corresponding DD (9) is odd in z . Hence, for small or moderate η values both GPDs (11) are approximately given by the corresponding chiral even GPDs.

MODEL PREDICTIONS FOR CHIRAL ODD GPDS

Assuming SU(6) symmetry, let us now provide some GPD predictions for the scalar diquark contents, which have the quark combination $2u/3 - d/3$. Taking the functional form of the reduced LCWF, which enters in the unintegrated DD (6), as

$$\phi^{\text{pow.}}(\bar{\alpha}) = \frac{g \bar{\alpha}^{p+\alpha/2}}{M\Gamma(1+p+\alpha/2)}, \quad \phi^{\text{exp.}}(\bar{\alpha}) = \frac{g}{M} \delta(\bar{\alpha} - \bar{A}), \quad (12)$$

and adopting a Regge inspired form for the spectral density [11]

$$\rho_\alpha(\lambda, \lambda_{\text{cut}}) = \theta(\lambda^2 - \lambda_{\text{cut}}^2) \frac{(\lambda^2 - \lambda_{\text{cut}}^2)^{\alpha-1}}{\Gamma(\alpha) M^{2\alpha}}, \quad (13)$$

we build three models, two with power-like and one with exponential \mathbf{k}_\perp fall-off. Here, g is the coupling of the struck quark to the spectator system, p determines the power-like fall-off of \mathbf{k}_\perp -dependence, α is taken to be close to the ρ Regge trajectory or, alternatively, set to zero, and \bar{A} determines the slope of the exponential \mathbf{k}_\perp -dependence. All these parameters together with the struck quark mass and the spectator threshold λ_{cut} are fitted to the functional shape of PDFs and form factors, where our Regge improved power-like model provides a good agreement. The GPD predictions for the eight twist-two GPDs at the cross-over line $\eta = x$ are shown in Fig. 1. Note that this quantity determines at leading order the imaginary part of hard exclusive amplitudes, where the real part can be obtained from a partonic dispersion relation. As one realizes, the chiral odd quantities, including the "pretzelosity" related \tilde{H}_T GPD, are rather sizeable.

CONCLUSIONS

Utilizing the underlying Lorentz symmetry through the DD, parton number conserved LCWF overlap representations are employed to model GPDs in the entire momentum fraction region. In a scalar diquark model one finds easily relations among GPDs which are the analogies to the relations among uPDFs. Moreover, the unintegrated DD given in (6) tells us that in such models t -dependency in GPDs and \mathbf{k}_\perp -dependency in uPDFs are tied to each other, however, a one-to-one correspondence can be obtained only when the DD is concentrated at one specific z value, e.g., $z = 0$. Finally, effective LCWF overlap representations may become a useful phenomenological tool to quantify the partonic contents in a global fitting procedure to both exclusive and inclusive observables.

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